Appendix A: Theoretical bidding model

In this section, we present a formal model of privatization competitions. The government requires the performances of certain functions, indexed by i. At the onset, in stage 0, function i is performed by an in-house team, labeled I. Let Y_{1i} denote the baseline cost of function i. Baseline cost is therefore the cost to the government of having the function performed during this initial stage. Baseline cost is determined by the following:

$$Y_{1i} = Y_1 (X_i, X_I, \Lambda_0, u_{1i}) \ , \tag{10}$$

where X_i is a vector of variables relating to the scale and complexity of function i, X_I is a vector of variables relating to IS inherent technological efficiency, Λ_0 is a vector of variables relating to the stringency of the government's monitoring and control of *IS* costs, and u_{1i} is an unobservable error term. To illustrate the role of A, in equation (1), holding all other variables constant, baseline cost may be relatively high if the government exerts little control over I's cost and relatively low if I under an optimal incentive contract along the lines proposed in [11]. In practice, variables X_I and Λ_0 may be unobservable and therefore folded into the error term for estimation purposes. Assuming this is so, and defining a log-linear relationship between the dependent and independent variables, the specification of (1) becomes (2)

$$\ell n \, (Y_{1i}) = X_i \beta_1 + u_{1i} . \tag{11}$$

In the next stage of the model, the government conducts a privatization competition such as the A-76 competition. In the privatization competition, the in-house team and a number of private contractors bid for the right to be the sole provider of function i for the government. Let $\{P_j\}$ be the set of private contractors that are potential bidders. This set includes actual participants in the privatization competition as well as those who elect not to participate, effectively submitting infinite bids. The players submit secret bids simultaneously. A private contractor's bid is the price at which it agrees to perform the function; the in-house team's bid is the cost at which it agrees to perform the function. We assume all players know their own cost of performing the function. Thus, the privatization competition is a private-values procurement auction.

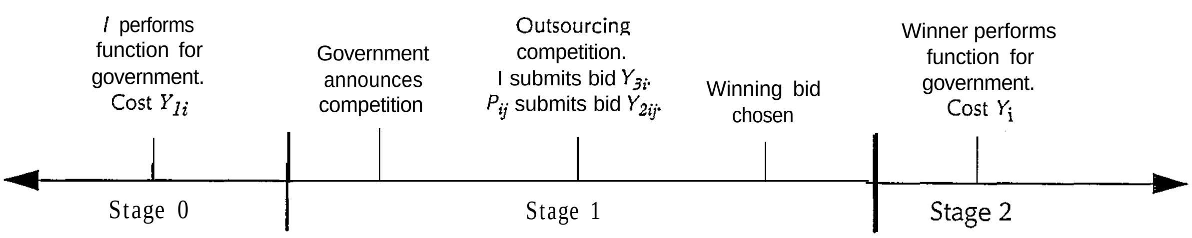
Let Y_{3i} be 19 bid. Let Y_{2i} be the lowest of the outside contractor's bids. That is, $Y_{2i} = min\{Y_{2ij} \mid j = 1, 2, \ldots N_i\}$, where Y_{2ij} is the bid of contractor P_j. The government selects the winning bid Y_i according to prespecified rules. A simple rule would be to select the lowest bid. We will allow the government to use a more complicated selection rule, possibly providing an incumbency advantage to the in-house team:

$$\begin{aligned} Y_i &= Y_3 \text{ if } Y_{3i} \leq (1 + A)\, Y_{2i} \\ &= Y_{2i} \text{ if } Y_{3i} > (1 + \Delta)\, Y_{2i} \end{aligned} \tag{12}$$

According to equation (12), *I* wins the competition as long as its bid is less than a scaled-up version of the private contractor's bid. For the A-76 competitions considered in this study, the scaling factor, A, was 10 percent. Of course, if A= 0, then equation (12) simply selects the lowest bid.

In the last stage of the game, the winning bidder performs the task for the government according to the terms of its bid. A schematic diagram of the timing of the game is provided in figure 4.

Figure 4. Timing of model



Private contractor's bids

Private contractor P_j is assumed to choose its bid Y_{2ij} to maximize its expected profit, defined as the payment from the government minus the cost of performing the task. An increase in its bid has two effects: the positive effect of increasing the revenue it obtains if it wins the outsourcing competition and the negative effect of decreasing the probability if it wins the competition. Formally, let $\Psi(X_i, X_j, \nu_j)$ denote P_j's cost of performing the task. This cost depends on several variables. As before, X_i is a vector of task-specific variables relating to the scale and complexity of the task that would increase the cost of performing. X_j is a vector of contractor-specific variables that would increase the cost of performing the task. X_i and X_j are assumed to be observable to all players. Similar to X_j, ν_j is a vector of variables relating to P_j's cost of performing the task, the difference being that ν_j is assumed to be unobservable to all players except P_j. Let f_j be the density function and F_j be the distribution function associated with ν_j. Assume these functions are common to all players. Normalizing its profit conditional on losing the outsourcing competition to zero, we can express P_j's optimization problem as

$$Y_{2ij} = \operatorname{argmax}_{C \in \Re^+} \{[C - \Psi(X_i, X_j, \nu_j)]\Pr[C < min(\{Y_{2ik}\}_{k \neq j}, [1/(1+\Delta)\,Y_{3i})]\}. \tag{13}$$

From equation (4), we can derive the following reduced-form expression for P_j's equilibrium bid:

$$Y_{2ij} = Y_{2j}\left(X_i, X_j, \{X_k\}_{k \neq j}, X_I, \Delta, \Lambda_0, \nu_j\right). \tag{14}$$

It is clear that Y_{2ij} will depend on X_i, X_j, and ν_j since these variables directly affect P_j's cost of performing the task. The other variables affect Y_{2ij} indirectly by affecting the best-response functions of the other players. Just as X_j affects P_j's bid, a vector of variables increasing P_k's cost of performing the task, denoted by X_k, affects P_k's bid. Similarly, a vector of variables increasing I's cost of performing the task, denoted by X_I, affects I's bid. An increase in any of the variables in $\{X_k | k \neq j\}$ or X_I will tend to cause P_j's rivals to bid less aggressively, and this will have the indirect effect of causing P_j to bid less aggressively.

Of course, this indirect effect can only occur if $\{X_k | k \neq j\}$ and X_I are observable to P_j, as we have assumed. Similarly, an increase in X_i will affect a rival's best-response functions and thus will have an indirect effect, as well as a direct effect, on P_j's equilibrium bid. As will be discussed in more detail in the next subsection, increases in Band Λ_0 may have an indirect effect on P_j's bid by causing I to bid less aggressively.[19]

In view of equation (14), the minimum of the private contractors' bids can be written

$$Y_{2i} = Y_2\left(X_i, \{X_j | j = 1, \ldots, N_i\}, X_I, \Delta, \Lambda_0, \{v_j | j = 1, \ldots, N_i\} \right) . \tag{15}$$

In practice, we may not observe variables $\{ X_j | j = 1, \ldots, N_i\}$, X_I or A, , Recognizing this fact, letting the relationship between the independent and dependent variables be linear, and collecting the unobservable error terms together as u_{2i}, equation (15) becomes (16) as shown below

$$Y_{2i} = X_i \beta_2 + u_{2i}. \tag{16}$$

Note that another difference between equations (15) and (16) is the exclusion of a Δ term. A has been excluded from equation *(16)* because, in practice, the rules of the privatization competitions held A constant across function i. It is important to remember that the coefficients β_2 are conditioned on A. Changes in A will cause changes in β_2. However, it may be possible to predict the direction of the effect of A on β_2 , so policy simulations involving changes in A will still be useful.

In-house team's bid

IS decision problem is similar to the private contractor's described above. One difference is that the private contractors are for-profit

19. The comparative-statics results given in this paragraph are conjectures meant to build intuition regarding the reduced-form bid in (12). In general, it is not possible to sign the partials unless strong assumptions are placed on the distribution of the unobservable terms ν_i and ν_I.

firms, while I is a nonprofit unit of government. We will address this first issue by supposing the cost-reduction provides I with disutility, rather than a reduction in revenues as with the private contractors. Another important difference is that at the privatization stage, I has already demonstrated it can provide the task at a cost Y_{1i} in stage 0. The government can reduce the ultimate cost of obtaining the task by constraining I's bid to lie below Y_{1i}. We will address this issue by treating l's desired bid as a latent variable that may differ because of the government's constraint from I's actual bid.

Let Y^*_{5i} be the solution to the following optimization problem for I:

$$Y^*_{3i} = \operatorname{argmax}_{C \in \Re^+} \{ U(C, X_i, X_I, \nu_I) \Pr[C < (1+\Delta) Y_{2i}] \} , \tag{17}$$

where X_I and ν_I are to I as X_j and ν_j are to P_j. To account for the fact that I is a nonprofit unit, instead of the profit term appearing in (4), a term measuring I's surplus from performing the task, denoted by $U(C, X_i, X_I, \nu_I)$, appears in (17). We assume $\partial U/\partial C > 0$. That is, we assume l's utility is lower for a lower cost target. There are many potential sources of this disutility: effort, the disutility of cost-cutting measures such as firing employees, and so forth. As a reduced form, Y^*_{3i} can be written

$$Y^*_{3i} = Y^*_3 (X_i, X_I, \{X_j | j=1, \ldots N_i\} \Delta, \Lambda_O, \nu_I) . \tag{18}$$

It is clear from (12) that Y^*_{3i} should depend on A, the bidding advantage given to I by the government. An increase in A will tend to make Ibid less aggressively. It may be less clear why Y^*_{3i} should depend on A,. This is due to an indirect effect. Though Y^*_{3i} may not depend directly on A,, as will be formalized below, its realized bid, Y_{3i}, may. The private contractors form their bids $\{ Y_{2ij} | j = 1, \ldots N_i\}$ based on their expectations of Y_{3i}, and Y^*_{3i} in turn depends on I's expectations of $\{ Y_{2ij} | j = 1, \ldots N_i\}$, This indirect effect will only arise if A, is observable to the private contractors. Similar reasoning can be offered that, in addition to its direct effect on Y^*_{3i}, X_i may exert an indirect effect on Y^*_{3i} as well.

By analogy to the derivation of (16) from (15), we can express (18) in a form that can be implemented as

$$Y^*_{3i} = X_i\beta_3 + u_{3i} \ . \tag{19}$$

The government's constraint that I's realized bid cannot exceed baseline cost implies

$$Y_{3i} = min(Y_{1i}, Y^*_{3i}) \ . \tag{20}$$

To interpret (20), Y^*_{3i} can be thought of as a latent variable measuring I's desired bid. If Y^*_{3i} is less than Y_{1i}, Y^*_{3i} is the realized bid. If Y^*_{3i} exceeds Y_{1i}, the latent variable is not observed, and Y_{1i} is the observed or realized bid.

Cost savings from competition

In this section, we derive an expression for the expected cost savings due to outsourcing. The discussion will be based on reduced-form equations (11), (16), and (19). Some simplifying notation will prove useful. Let $u_i = (u_{1i}, u_{2i}, u_{3i})$ be the vector of error terms and f be its associated density. Define U_i to be a set of realizations of u_i, U_{Ii} to be the subset of u_i such that I wins the outsourcing competition, and U_{Pi} the subset of u_i such that a private contractor wins. That is,

$$U_{Ii} = \{u_i \in U_i | Y_{3i} \leq (1 + \Delta)\, Y_{2i}\}$$

$$U_{Pi} = \{u_i \in U_i | Y_{3i} > (1 + \Delta)\, Y_{2i}\} \ .$$

The expected cost savings from the competition equals

$$\int_{U_i} Y_{1i} f(u_i)\, du_i - \int_{U_{Pi}} Y_{2i} f(u_i)\, du_i - \int_{U_{Ii}} Y_{3i} f(u_i)\, du_i \ . \tag{21}$$

Note that equation (21) is a highly nonlinear function of the errors. In general, there exists no easily attainable closed-form solution for this expectation. This justifies the use of the simulation methodology for predicting savings discussed in the text.

Appendix B: Estimating the bidding and baseline cost equations

Consider the three equation models given by

$$\ell n\,(Y_{1i}) = X_i\beta_1 + u_{1i} \tag{22}$$

$$\ell n\,(Y_{2i}) = X_i\beta_2 + u_{2i} \tag{23}$$

$$\ell n\,(Y^*_{3i}) = X_i\beta_3 + u_{3i}\ , \tag{24}$$

where

$$Y_{3i} = \min\,(Y^*_{3i}, Y_{1i})\ , \tag{24a}$$

Equation (23) can be estimated consistently with ordinary least squares. Equations (22) and (24) can be estimated consistently with a maximum likelihood (ML) procedure.

To estimate equations (22) and (24), assume the vector of error terms $u_i = (u_{1i}, u_{2i}, u_{3i})$ is distributed Normally with zero mean and covariance matrix Σ given by

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix} .$$

Defining $\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$, the likelihood function for the joint estimation of equations (22) and (24) is given by

$$L = \Pi\{i|Y_{3i}^* < Y_{1i}\}\, Pr(u_{1i})\, Pr(u_{3i}|u_{1i})\, \Pi\{i|Y_{3i}^* \geq Y_{1i}\}\, Pr(u_{1i})\, Pr(Y_{3i}^* \geq Y_{1i}|u_{1i}) \tag{25}$$

where $Pr(u_{1i})$, $Pr(u_{3i}|u_{1i})$, and $Pr(Y_{3i}^* \geq Y_{1i}|u_{1i})$ are given by

$$Pr(u_1) = \frac{1}{\sqrt{2\pi\sigma_1}} \exp\left[-\frac{1}{2}\left(\frac{u_{1i}}{\sigma_2}\right)^2\right] = \frac{1}{\sigma_1}\phi(u_{1i}) \quad . \tag{26}$$

$$Pr(u_{3i}|u_{1i}) = \frac{1}{\sqrt{2\pi\sigma_3^2(1-\rho_{13}^2)}} \exp\left[-\frac{1}{2\sigma_3^2(1-\rho_{13}^2)}\left(u_{3i} - \frac{\rho_{13}\sigma_3}{\sigma_1}u_{1i}\right)^2\right]$$

$$= \frac{1}{\sigma_3\sqrt{1-\rho_{13}^2}}\,\phi\left(\frac{u_{3i} - \frac{\rho_{13}\sigma_3}{\sigma_1}u_{1i}}{\sigma_3\sqrt{1-\rho_{13}^2}}\right) \tag{27}$$

$$Pr(Y_{3i}^* \geq Y_{1i}|u_{1i}) = Pr(X_i\beta_3 + u_{3i} \geq Y_{1i}|u_{1i})$$

$$= Pr(u_{3i} \geq Y_{1i} - X_i\beta_3|u_{1i})$$

$$= 1 - \Phi\left(\frac{u_{3i} - \frac{\rho_{13}\sigma_3}{\sigma_1}u_{1i}}{\sigma_3\sqrt{1-\rho_{13}^2}}\right) , \tag{28}$$

where

ϕ = Standard normal probability density function

Φ = Standard normal cumulative distribution function.

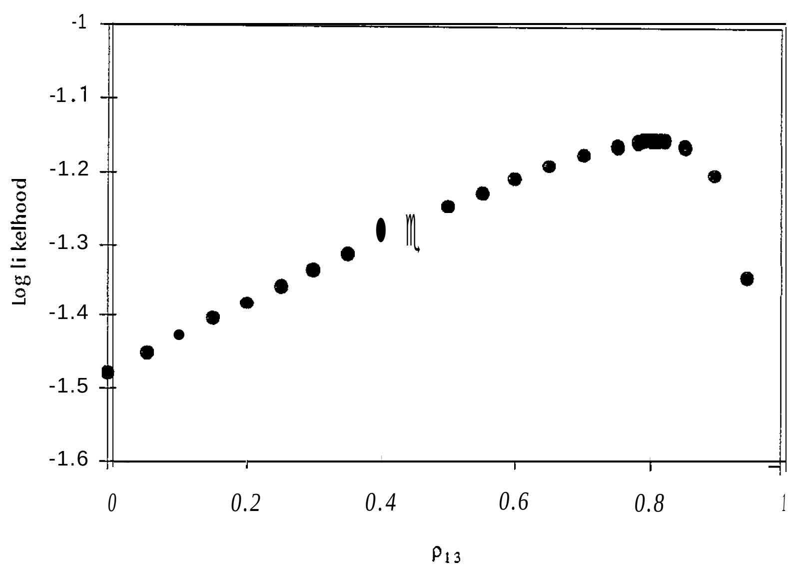
Substituting $Pr(u_{1i})$, $Pr(u_{3i}|u_{1i})$, and $Pr(Y_{3i}^* \geq Y_{1i}|u_{1i})$ in equation (4) gives

$$L = \prod_{\{i \mid Y^*_{3i} < y_{1i}\}} \left(\frac{1}{\sigma_1}\phi\,(Y_{1i} - X_i\beta_1)\right)\left[\frac{1}{\sigma_3\sqrt{1-\rho^2}}\phi\left(\frac{Y_{3i} - X_i\beta_3 - \dfrac{\rho_{13}\sigma_3}{\sigma_1}(Y_{1i} - X_i\beta_1)}{\sigma_3\sqrt{1-\rho_{13}^2}}\right)\right]$$

$$\times \prod_{\{i \mid Y^*_{3i} \geq Y_{1i}\}} \left(\frac{1}{\sigma_1}\phi\,(Y_{1i} - X_i\beta_1)\right)\left[1 - \Phi\left(\frac{Y_{3i} - X_i\beta_3 - \dfrac{\rho_{13}\sigma_3}{\sigma_1}(Y_{1i} - X_i\beta_1)}{\sigma_3\sqrt{1-\rho_{13}^2}}\right)\right]. \tag{29}$$

Olsen in [12] has shown that likelihood functions such as equation (29) have a unique maximum conditional on ρ. Also, Nawata in [13] has shown that the likelihood function conditional on ρ gives reliable estimates. For this reason, we maximize equation (29) for a given $\hat{\rho}_{13}$ and then search over the interval $-0.99 \leq \hat{\rho}_{13} \leq 0.99$ for the final ML estimates. A plot of the maximum likelihood value as a function of $\hat{\rho}_{13}$ is given in figure 5.

Figure 5. Grid search over ρ_{13}



Note that all the element of Σ can be easily estimated except σ_{23}. Estimates of ρ_{13}, σ_1^2, and σ_3 are obtained by the ML procedure which can be used to also compute σ_{13}. An estimate of σ_2^2 is obtained from OLS, and an estimate of ρ_{12} can be obtained from within the sample residuals of equations (22) and (23). An estimate of ρ_{23} can be obtained with stochastic simulation of predicted savings from

$$Y_{1i} = \exp(X_i\beta_1 + u_{1i}) \tag{30}$$

$$Y_{2i} = \exp(X_i\beta_2 + u_{2i}) \tag{31}$$

$$Y_{3i} = min[\exp((X_i\beta_3 + u_{3i}), Y_{1i})] \tag{32}$$

$$\begin{aligned} S_i &= Y_{1i} - Y_{3i} \text{ if } Y_{3i} \leq (1 + A)\, Y_{2i} \\ &= Y_{1i} - Y_{2i} \text{ if } Y_{3i} > (1 + A)\, Y_{2i} \end{aligned} \tag{33}$$

Let u_j be the j^{th} draw from a normal distribution with covariance matrix Σ, where the estimate of ρ_{23} is set at -0.99. Substituting u_j and the parameter estimates into equations (30)–(33) yields the j^{th} draw of savings for the i^{th} function in the A-‘76 completed competitions, denoted by S_{ij}. If this process is repeated R times with R separate draws of u_j, an estimate of savings for a completed A-76 competition for funciton i is

$$S_i = \frac{1}{R}\sum_{j=1}^{R} S_{ij} . \tag{34}$$

This process can be repeated for each of the N functions in the completed A-76 competitions. Total predicted savings for the N completed A-76 competitions is given by

$$Stot = \sum_{i=1}^{N} S_i . \tag{35}$$

To obtain an estimate of ρ_{23}, repeat the stochastic simulation of equations (30) to (35) in 0.01 step increments for all values of $\hat{\rho}_{23}$ in the interval -0.99 to 0.99. The value of $\hat{\rho}_{23}$ that gives predicted savings from equation (35) equal to observed savings in the completed A-‘76 competitions is the estimate of ρ_{23}.

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